

(科目:) 数 学 作 业 纸

编号:

班级:

姓名:

第 页

总结一个

电荷: $\rho = \rho_f + \rho_p = \rho_f - \nabla \cdot \vec{P}$

电流: $\vec{J} = \vec{J}_f + \vec{J}_m + \vec{J}_p = \vec{J}_f + \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t}$, $\vec{J}_f = \vec{J}_c + \vec{J}_e$

Maxwell:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \cdot \vec{B} = 0$$

边界:

$$\hat{n} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_k$$

$$\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$$

$$\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

线性、各向同性、非色散: $\vec{B} = \mu \vec{H}$, $\vec{D} = \epsilon \vec{E}$, $\vec{J}_c = \sigma \vec{E}$

色散: $\vec{D}(t) = \epsilon_0 \int_{-\infty}^t \epsilon_r(t-t') \vec{E}(t') dt'$, $\vec{B}(t) = \mu_0 \int_{-\infty}^t \mu_r(t-t') \vec{H}(t') dt'$, 因果性.

Poynting: $-\nabla \cdot \vec{S} = \frac{\partial w_e}{\partial t} + \frac{\partial w_m}{\partial t} + p_d + p_s$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$w_e = \frac{1}{2} \vec{E} \cdot \vec{D}, w_m = \frac{1}{2} \vec{B} \cdot \vec{H}$$

$$p_d = \sigma \vec{E}^2, p_s = \vec{J}_e \cdot \vec{E}$$

时谐场/源下的Maxwell:

$$\nabla \times \vec{E} = i\omega \vec{B}$$

$$\nabla \times \vec{H} = \vec{J}_f - i\omega \vec{D}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \cdot \vec{B} = 0$$

线性、各向同性、色散/非色散: $\vec{D}(\omega) = \epsilon(\omega) \vec{E}(\omega)$, $\vec{B}(\omega) = \mu(\omega) \vec{H}(\omega)$

Poynting: $-\nabla \cdot \langle \vec{S} \rangle = 2i\omega(w_e - w_m) + \frac{1}{2} \sigma \vec{E} \cdot \vec{E}^* + \frac{1}{2} \vec{J}_e^* \cdot \vec{E}$ $\langle \vec{S} \rangle = \frac{1}{2} \vec{E} \times \vec{H}^*$

$$w_e = \frac{1}{4} \vec{E} \cdot \vec{D}^*, w_m = \frac{1}{4} \vec{B} \cdot \vec{H}^*$$

$$-\nabla \cdot \langle \vec{S} \rangle = \frac{1}{2} \omega \epsilon'' |\vec{E}|^2 + \frac{1}{2} \omega \mu'' |\vec{H}|^2 + \frac{1}{2} \sigma |\vec{E}|^2 + \text{Re} \left\{ \frac{\vec{J}_e^* \cdot \vec{E}}{2} \right\} \quad \langle \vec{S} \rangle = \frac{1}{2} \text{Re} \{ \vec{E} \times \vec{H}^* \}$$

Maxwell 方程组唯一性 业 补 学 题 (日 体)

$\left. \begin{array}{l} \text{区域内 } t=0 \text{ 时 } \vec{E}, \vec{H} \text{ 已知} \\ \text{边界上 } t>0 \text{ 时 } \vec{E}|_S, \vec{H}|_S \text{ 已知} \end{array} \right\} \text{区域内 } \vec{E}(\vec{r}, t), \vec{H}(\vec{r}, t) \text{ 唯一-确定.}$

$\Rightarrow \left\{ \begin{array}{l} \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{H} = \vec{j}_0 + \nabla \times \vec{E} + \frac{\partial \vec{D}}{\partial t} \\ \nabla \cdot \vec{D} = \rho \\ \nabla \cdot \vec{B} = 0 \end{array} \right.$

假设 $(\vec{E}_1, \vec{H}_1), (\vec{E}_2, \vec{H}_2)$ 是两解

令 $\vec{E} = \vec{E}_2 - \vec{E}_1$

$\vec{H} = \vec{H}_2 - \vec{H}_1$
 得到

$-\oint (\vec{E} \times \vec{H}) \cdot d\vec{s} = \frac{d}{dt} \int_V (\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2) dV + \int_V \sigma \vec{E}^2 dV$

$t > 0 \text{ 时, } \vec{E}|_S = 0, \vec{H}|_S = 0 \Rightarrow \oint (\vec{E} \times \vec{H}) \cdot d\vec{s} = 0$

$\therefore \frac{d}{dt} \int_V (\frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2) dV = - \int_V \sigma \vec{E}^2 dV \leq 0$

$t = 0 \text{ 时, } \frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 = 0$

$\therefore \frac{1}{2} \epsilon \vec{E}^2 + \frac{1}{2} \mu \vec{H}^2 = 0 \Rightarrow \vec{E} = 0, \vec{H} = 0$

$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = -\nabla \cdot \vec{j}_0$
 $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = -\nabla \cdot \vec{j}_0$
 $\nabla \cdot \vec{j} = -\nabla \cdot \vec{j}_0$
 $0 = \nabla \cdot \vec{j}$

$\vec{H} \times \vec{E} = \vec{E}$

$\vec{j} = \nabla \times \vec{A} + \frac{\partial \vec{A}}{\partial t} + \vec{j}_0 = \vec{E} - \vec{j}_0$

$\vec{H} \cdot \vec{E} = \omega, \vec{E} \cdot \vec{E} = \omega$

$\vec{E} \cdot \vec{j} = 0, \vec{E} \cdot \vec{j}_0 = \omega$

Maxwell 方程组唯一性证明

$\vec{j} = \nabla \times \vec{A} + \frac{\partial \vec{A}}{\partial t} + \vec{j}_0$

$\nabla \cdot \vec{j} = -\nabla \cdot \vec{j}_0$

$\nabla \cdot \vec{j} = 0$

$\vec{j} = \vec{j}_0$

Maxwell 方程组唯一性证明

$\vec{j} = \nabla \times \vec{A} + \frac{\partial \vec{A}}{\partial t} + \vec{j}_0 = \vec{E} - \vec{j}_0$

$\vec{H} \cdot \vec{E} = \omega, \vec{E} \cdot \vec{E} = \omega$

$\vec{E} \cdot \vec{j} = 0, \vec{E} \cdot \vec{j}_0 = \omega$

(科目:) 数 学 作 业 纸

编号:

班级:

姓名:

第 页

二、真空或介质中:

$$\text{解得} \begin{cases} \nabla^2 \vec{E} + k^2 \vec{E} = 0 \\ \nabla \cdot \vec{E} = 0 \\ \vec{B} = -\frac{i}{\omega} \nabla \times \vec{E} \end{cases}$$

$$\text{或} \begin{cases} \nabla^2 \vec{B} + k^2 \vec{B} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \vec{E} = \frac{i}{\omega \mu \epsilon} \nabla \times \vec{B} \end{cases}$$

均匀平面波解:

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \vec{E}_{\text{real}} = \text{Re}\{\vec{E}\} \quad \vec{H} = \frac{1}{\omega \mu} \vec{k} \times \vec{E} = \frac{1}{\eta} \hat{k} \times \vec{E} \quad (\eta = \sqrt{\frac{\mu}{\epsilon}} \approx 120\pi \approx 377 \Omega)$$

均匀: 等振幅面...

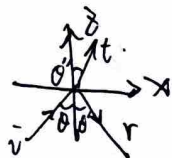
平面: 等相位面...

横波: $\hat{k} \cdot \vec{E} = 0$

$$v_p (= \frac{\omega}{k}) = v_g (= \frac{d\omega}{dk}) = v_e (= \frac{\langle \vec{S} \rangle}{\langle w_m \rangle})$$

$$\langle w_e \rangle = \langle w_m \rangle = \frac{1}{4} \epsilon |\vec{E}_0|^2, \quad \langle \vec{S} \rangle = \frac{|\vec{E}_0|^2}{2\eta} \hat{k}, \quad w_e = w_m = \frac{1}{2} \epsilon E^2, \quad \vec{S} = \frac{E^2}{\eta} \hat{k}$$

三. 介质表面上



$$\vec{k} \cdot \vec{x} = \vec{k}' \cdot \vec{x} = \vec{k}'' \cdot \vec{x}, \quad k_x = k'_x = k''_x$$

$$k_x = k \sin \theta, \quad k'_x = k' \sin \theta', \quad k''_x = k'' \sin \theta''$$

$$k' = k = \frac{\omega}{v_1}, \quad k'' = \frac{\omega}{v_2}$$

$$\therefore \theta = \theta' > \frac{\sin \theta}{\sin \theta''} = \frac{v_1}{v_2} = n_{21}$$

N波 (垂直极化)

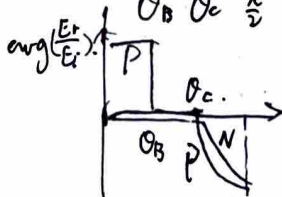
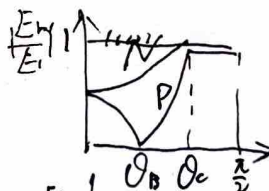
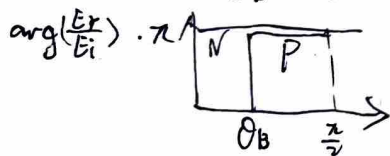
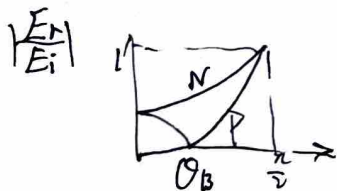
$$\begin{cases} E + E' = E'' \\ H \cos \theta - H' \cos \theta' = H'' \cos \theta'' \\ H = \sqrt{\frac{\epsilon}{\mu}} E \end{cases} \text{各媒质}$$

P波 (平行极化)

$$\begin{cases} E \cos \theta - E' \cos \theta' = E'' \cos \theta'' \\ H + H' = H'' \\ H = \sqrt{\frac{\epsilon}{\mu}} E \end{cases} \text{各媒质}$$

光疏 \rightarrow 光密

光密 \rightarrow 光疏



讨论:

① Brewster: $\theta_B = \arctan(n_2/n_1)$: P波全折射

② 相位损失: N波: 光疏 \rightarrow 光密.

P波: 光疏 \rightarrow 光密 $\theta > \theta_B$, 光密 \rightarrow 光疏 $\theta < \theta_B$.

③ 全反射

$$k_z'' = \sqrt{k''^2 - k_x^2} = ik \sqrt{\sin^2 \theta - n_{21}^2}, \quad \text{令 } K = k \sqrt{\sin^2 \theta - n_{21}^2} \text{ 则:}$$

$$1) \vec{E}'' = \vec{E}_0'' e^{i(k_x x - \omega t)} = \vec{E}_0'' e^{-Kz} e^{i(k_x x - \omega t)} \quad \text{非均匀平面波}$$

$$v_p'' = \frac{\omega}{k_x''} < \frac{\omega}{k''} \quad (k_x'' > k'') \Rightarrow \text{慢波}$$

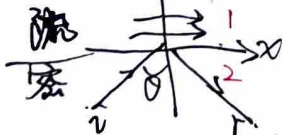
$$2) \vec{E} + \vec{E}' = 2\vec{E}_0 \cos(k_x z + \phi) e^{i(k_x x - \omega t - \phi)} \quad \text{非均匀平面波}$$

$$v_p = \frac{\omega}{k_x} > \frac{\omega}{k} \quad (k_x < k) \Rightarrow \text{快波}$$

其中, $E/E = e^{-2i\phi}$, $\phi = \begin{cases} \arctan \frac{\sqrt{\sin^2 \theta_1 - n_{21}^2}}{\cos \theta_1} \\ \arctan \frac{\sqrt{\sin^2 \theta_1 - n_{21}^2}}{n_{21} \cos \theta_1} \end{cases}$

N $\therefore k_x < k < k'' < k_x''$

P $\therefore v_p'' < v_p < v_p' < v_p$



(科目:) 数 学 作 业 纸

编号:

班级:

姓名:

第 页

四. 导体中.

$$\text{解得 } \begin{cases} \nabla^2 \vec{E} + \omega^2 \mu \tilde{\epsilon} \vec{E} = 0 \\ \nabla \cdot \vec{E} = 0 \end{cases}$$

$$\tilde{\epsilon} = \epsilon + i\frac{\sigma}{\omega}$$

$$k^2 = \omega^2 \mu \tilde{\epsilon} = (\vec{\beta} + i\vec{\alpha})^2$$

均匀平面波解:

$$\vec{E} = \vec{E}_0 e^{-\vec{\alpha} \cdot \vec{x}} e^{i(\vec{\beta} \cdot \vec{x} - \omega t)}$$

$$\vec{E}_{\text{real}} = \text{Re}\{\vec{E}\}$$

$$\vec{H} = \frac{1}{\omega \mu} (\vec{\beta} + i\vec{\alpha}) \times \vec{E}$$

$$\begin{cases} \text{均匀: } e^{-\vec{\alpha} \cdot \vec{x}} \\ \text{平面: } \vec{\beta} \cdot \vec{x} - \omega t \end{cases}$$

$$(\vec{\beta} + i\vec{\alpha}) \cdot \vec{E} = 0 \quad \vec{\beta} \cdot \vec{E} \neq 0$$

(科目:) 数 学 作 业 纸

编号:

班级:

姓名:

第 页

良导体表面上.

$$\frac{\sigma}{\omega} \gg 1.$$



$$k_x = k'_x = \beta_x + i\alpha_x \Rightarrow \alpha_x = 0, \beta_x = k_x$$

$$k^2 = \beta^2 - \alpha^2 + 2i\vec{\alpha} \cdot \vec{\beta} = \omega^2 \mu (\epsilon + i\frac{\sigma}{\omega}) \approx i\omega\mu\sigma.$$

$$\therefore \beta^2 - \alpha^2 \approx 0, \vec{\alpha} \cdot \vec{\beta} = \alpha_2 \beta_2 = \frac{1}{2} \omega\mu\sigma = \frac{1}{2} \omega\mu\epsilon_0 \left(\frac{\sigma}{\omega\epsilon_0}\right) \gg \frac{1}{2} k^2 > \frac{1}{2} k_x^2.$$

$$\therefore \alpha_2 \approx \beta_2 \approx \sqrt{\frac{\omega\mu\sigma}{2}}, \beta_x \ll \beta_2, \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}.$$

$$\therefore \vec{H} = \frac{1}{\omega\mu} (\beta_2 + i\alpha_2) \hat{n} \times \vec{E} \approx \sqrt{\frac{\sigma}{\omega\mu}} e^{i\frac{\pi}{4}} \hat{n} \times \vec{E}. \quad \text{H 滞后 E } \frac{\pi}{4}$$

$$\left\{ \begin{aligned} \sqrt{\frac{\mu}{\epsilon}} |\vec{H}| &= \sqrt{\frac{\sigma}{\omega\epsilon}} \gg 1 \end{aligned} \right.$$

磁场能 \Rightarrow 电场能.

N波. 垂直入射

$$\left\{ \begin{aligned} E + E' &= E'' \\ H - H' &= H'' \\ H &= \sqrt{\frac{\epsilon_0}{\mu_0}} E, H' = \sqrt{\frac{\epsilon_0}{\mu_0}} E' \\ H'' &= \sqrt{\frac{\sigma}{\omega\mu}} e^{i\frac{\pi}{4}} E'' \end{aligned} \right.$$

P波. 垂直入射.

$$\left\{ \begin{aligned} E - E' &= E'' \\ H + H' &= H'' \\ H &= \sqrt{\frac{\epsilon_0}{\mu_0}} E, H' = \sqrt{\frac{\epsilon_0}{\mu_0}} E' \\ H'' &= \sqrt{\frac{\sigma}{\omega\mu}} e^{i\frac{\pi}{4}} E'' \end{aligned} \right.$$

$$R = \left| \frac{E'}{E} \right|^2 \approx 1 - 2\sqrt{\frac{2\omega\epsilon_0}{\sigma}}.$$

讨论: 良导体内的电磁波.

$$\vec{E} = \vec{E}_0 e^{-\alpha z} e^{i(\beta z - \omega t)}.$$

$$\vec{H} = \sqrt{\frac{\sigma}{\omega\mu}} e^{i\frac{\pi}{4}} \hat{n} \times \vec{E}$$

$$\vec{S} = \frac{1}{2} \text{Re} \{ \vec{E} \times \vec{H}^* \} = \frac{1}{2} \sqrt{\frac{\sigma}{2\omega\mu}} E_0^2 e^{-2\alpha z} \hat{n}, S|_{z=0} = \frac{1}{2} \sqrt{\frac{\sigma}{2\omega\mu}} E_0^2.$$

$$P = \frac{1}{2} \text{Re} \{ \vec{J} \cdot \vec{E}^* \} = \frac{1}{2} \text{Re} \{ \sigma \vec{E} \cdot \vec{E}^* \} = \frac{1}{2} \sigma E_0^2 e^{-2\alpha z}, P_z = \int_0^\infty P dz = S|_{z=0}.$$

$$\vec{J}_s = \hat{n} \times \vec{H} = -\sqrt{\frac{\sigma}{\omega\mu}} e^{i\frac{\pi}{4}} \vec{E} = -\vec{E}_0 \sqrt{\frac{\sigma}{\omega\mu}} e^{-\alpha z} e^{i(\beta z - \omega t)} e^{i\frac{\pi}{4}}, P_s = \frac{1}{\sigma} R, \text{ 则 } P_z = \frac{1}{2} |\vec{J}_s|^2 P_s.$$

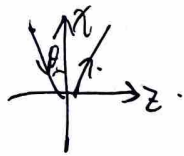
编号:

班级:

姓名:

第 页

六. 理想导体表面. $\sigma \rightarrow \infty$ & $\delta \rightarrow 0$.



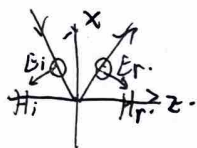
$$\hat{n} \times \vec{E} = 0$$

$$\hat{n} \times \vec{H} = \vec{J}_s$$

$$\hat{n} \cdot \vec{D} = \rho_s$$

$$\hat{n} \cdot \vec{B} = 0$$

N波:



$$E_{io} = -E_{ro} = E_0$$

$$\vec{E}_i = E_0 e^{i(k_x x + k_z z - \omega t)} \hat{e}_y$$

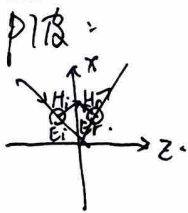
$$\vec{E}_r = -E_0 e^{i(k_x x + k_z z - \omega t)} \hat{e}_y$$

$$\therefore \vec{E}_t = \vec{E}_i + \vec{E}_r = -2i E_0 e^{i(k_z z - \omega t)} \sin k_x x \hat{e}_y$$

$$\begin{aligned} \therefore \vec{H} &= \frac{1}{i\omega\mu} \nabla \times \vec{E} = \frac{1}{i\omega\mu} (-2i E_0 e^{i(k_z z - \omega t)} k_x \cos k_x x \hat{e}_z + 2i E_0 i k_z e^{i(k_z z - \omega t)} \sin k_x x \hat{e}_x) \\ &= \frac{E_0 e^{i(k_z z - \omega t)}}{i\omega\mu} (-2i k_x \cos k_x x \hat{e}_z - 2k_z \sin k_x x \hat{e}_x) \\ &= \frac{i E_0 e^{i(k_z z - \omega t)}}{\omega\mu} (2k_z \sin k_x x \hat{e}_x + 2i k_x \cos k_x x \hat{e}_z) \end{aligned}$$

$$\therefore \rho_s = \hat{n} \cdot \epsilon_0 \vec{E} |_{x=0} = 0$$

$$\therefore \vec{J}_s = \hat{n} \times \vec{H} |_{x=0} = \frac{2k_x E_0 e^{i(k_z z - \omega t)}}{\omega\mu} \cos k_x x \hat{e}_y \Rightarrow -\frac{2k_x E_0 \cos k_x x e^{i(k_z z - \omega t)}}{\omega\mu} \hat{e}_y$$



$$\vec{E}_i = (E_x \hat{e}_x + E_z \hat{e}_z) e^{i(k_x x + k_z z - \omega t)}$$

$$\vec{E}_r = (E_x \hat{e}_x - E_z \hat{e}_z) e^{i(k_x x + k_z z - \omega t)}$$

$$\therefore E_{iz} = -E_{rz} = E_z$$

$$\therefore E_{ix} = E_{rx} = E_x$$

$$\therefore \vec{E} = \vec{E}_i + \vec{E}_r = 2E_x \cos k_x x e^{i(k_z z - \omega t)} \hat{e}_x - 2i E_z \sin k_x x e^{i(k_z z - \omega t)} \hat{e}_z$$

$$\therefore \vec{H} = \frac{1}{i\omega\mu} \nabla \times \vec{E} = \frac{2}{\omega\mu} (k_z E_x + k_x E_z) \cos k_x x e^{i(k_z z - \omega t)} \hat{e}_y$$

$$\therefore \rho_s = 2\epsilon_0 E_x \cos(k_x x - \omega t)$$

$$\vec{J}_s = \frac{2}{\omega\mu} (k_z E_x + k_x E_z) \cos(k_x x - \omega t) \hat{e}_y$$

无源 \rightarrow 有源

(科目:) 数 学 作 业 纸

编号:

班级:

姓名:

第 页

4. 电磁辐射

$$\begin{cases} \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases} \quad \begin{cases} \vec{A} \rightarrow \vec{A} + \nabla\psi \\ \phi \rightarrow \phi - \frac{\partial \psi}{\partial t} \end{cases}$$

库仑规范: $\nabla \cdot \vec{A} = 0$ (静磁、量子...)

$$\begin{cases} \nabla^2 \phi = -\frac{\rho}{\epsilon_0} \\ \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{j}_t \end{cases}$$

洛伦兹规范: $\nabla \cdot \vec{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$ (辐射、相对论)

$$\begin{cases} \nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon_0} \\ \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{j} \end{cases} \Rightarrow \begin{cases} \phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{x}', t_r)}{r} dV' \\ \vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{x}', t_r)}{r} dV' \end{cases} \Rightarrow \begin{cases} \vec{E} = \frac{1}{4\pi\epsilon_0} \left[\frac{[\rho]}{r^2} \hat{r} + \frac{[\dot{\rho}]}{cr} \hat{r} - \frac{[\ddot{\rho}]}{c^2 r} \hat{r} \right] dV' \\ \vec{B} = \frac{\mu_0}{4\pi} \left(\frac{[\dot{j}]}{r^2} + \frac{[\ddot{j}]}{cr} \right) \times \hat{r} dV' \end{cases}$$

时谐源场

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{x}')}{r} e^{ikr} dV'$$

远处观察: $r \gg \lambda$ (系统尺度, 积分范围度量)

$\lambda \sim l$ eg 半波天线 $kr \gg 2\pi$

eg 天线阵 $\begin{matrix} \downarrow & \downarrow \\ \vec{k} & \vec{a} \end{matrix}$ - 方向加强, - 方向减弱。

$\lambda \gg l$: $\lambda \gg r$: Near (static) zone: $e^{ikr} \sim 1$ 静电场 $\sim \frac{1}{r^2}$

$\lambda \ll r$: Far (radiation) zone: 辐射场 $\sim \frac{1}{r}$

$$\begin{aligned} \vec{A}(\vec{x}) &\approx \frac{\mu_0}{4\pi} \int \frac{\vec{j}(\vec{x}') e^{ik(\vec{R} - \hat{n} \cdot \vec{x}')}}{R - \hat{n} \cdot \vec{x}'} dV' \\ &\approx \frac{\mu_0}{4\pi} \left(\frac{[\vec{j}]}{R} + \frac{[\dot{\vec{j}}] \times \hat{n}}{cR} + \frac{\hat{n} \cdot [\ddot{\vec{j}}]}{c^2 R} + \dots \right) \end{aligned}$$

电偶极辐射: eg 短天线 $R \gg \lambda \gg l$
磁偶极辐射: eg 电流线圈 $R \gg \lambda \gg l$ 更小 $\vec{p} \rightarrow \frac{\vec{m}}{c} \times \vec{E} \times \vec{B} \rightarrow \frac{\vec{E}}{c}$

静源 \rightarrow 动源

(科目:) 数 学 作 业 纸

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第 页

八. 带电粒子辐射

$$\phi = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r - \frac{r \cdot \vec{v}}{c}} \right]$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{q[\vec{v}]}{[r - \frac{r \cdot \vec{v}}{c}]}$$

$$\vec{E} = \vec{E}_v + \vec{E}_a$$

$$\vec{B} = \vec{B}_v + \vec{B}_a$$

速度场: $\propto \frac{1}{r} \Rightarrow$ 不辐射

加速度场 $\propto \frac{1}{r} \Rightarrow$ 辐射

讨论:

$v \ll c$: 电偶极辐射.

$$\vec{p} = e\vec{x}_0 \quad f \rightarrow \vec{a}$$

$v \sim c$

$\vec{v} \parallel \vec{a}$: 韧致

eg 软X射线, 加速器

$\vec{v} \perp \vec{a}$: 同步

eg 加速器, 耗光大.

